Integrate the GM(1,1) and Verhulst Models to Predict Software Stage Effort

Yong Wang, Qinbao Song, Stephen MacDonell, Martin Shepperd, and Junyi Shen

Abstract—*Background:* Software effort prediction clearly plays a crucial role in software project management. *Problem:* In keeping with more dynamic approaches to software development, it is not sufficient to only predict the whole-project effort at an early stage. Rather, the project manager must also dynamically predict the effort of different stages or activities *during* the software development process. This can assist the project manager to reestimate effort and adjust the project plan, thus avoiding effort or schedule overruns. *Method:* This paper presents a method for software physical time stage-effort prediction based on grey models GM(1,1) and Verhulst. This method establishes models dynamically according to particular types of stage-effort sequences, and can adapt to particular development methodologies automatically by using a novel grey feedback mechanism. *Result:* We evaluate the proposed method with a large-scale real-world software engineering dataset, and compare it with the linear regression method and the Kalman filter method, revealing that accuracy has been improved by at least 28% and 50%, respectively. *Conclusion:* The results indicate that the method can be effective and has considerable potential. We believe that stage predictions could be a useful complement to whole-project effort prediction methods.

Index Terms—Grey prediction, software project management, software project stage-effort prediction.

I. INTRODUCTION

ACCURATE and unbiased software effort prediction is an important contributor to effective software project management [1]–[7]. Whole-project effort prediction is clearly important in terms of enabling software developers/managers to make a reasonable bid or form a plan of activities—consequently, an extensive body of research has addressed this facet of project management (see, for example, [1], [4], [5], and [8]–[11]). On the other hand, the capability to predict the effort required in different stages during the software development process is also important but rather less studied [3], [10], [12]–[16]. Such a capability enables (or at least should enable) project managers to identify potential effort overrun risk *during the project* and reallocate resources when necessary. It has been estimated that around 75% of all projects overrun their schedules due to inaccurate effort prediction [5]. The development and use of software stage-effort prediction methods have the potential to ensure that predictions are revisited and revised on an ongoing basis, and the result being that the accuracy of predictions should improve.

This is not to say, however, that the accurate prediction of software stage effort is straightforward. The software development process can proceed in a sporadic manner despite the best-laid plans. It is influenced by many uncertain and challenging-to-measure factors, such as individuals' levels of expertise, project difficulty, and technical complexity [3], [14], [17], [18]. Moreover, software development is largely a continuous and cumulative process. The work of prior stages forms the basis of current and subsequent stages, and there is evidence to suggest that there are some inherent relationships between the effort of prior and subsequent stages [3], [10], [12], [13]—although the nature of those relationships is not yet clear. This lends motivation to the idea of using prior stage effort to predict subsequent stage effort, a prediction process that should be continuous and dynamic in line with contemporary approaches to software development.

The stage-effort prediction problem has been addressed using a variety of data analysis methods (see Section II)—among them, linear regression (LR) analysis has been popular. In some cases, however, the relationships between different process stages was found to be not particularly strong, leading to some instances of large prediction error. In particular, predictions made in the early stages of a project tend to be more challenging—later-stage predictions can leverage the greater certainty that accrues with progress. Early-stage prediction must be performed in the context of data starvation, a context that is not conducive to commonly employed statistical methods (such as regression and those based on time series) and machine learning methods that usually require large data samples to determine statistical features of the series to build prediction models.

Grey system theory (GST), a system engineering theory based on the uncertainty of small samples, was first proposed by Deng in 1982 [19]. In keeping with the notion of a black box representing a system whose internal workings are not visible, a system about which we only know *some* information (mechanism, relationship, structure, connotation, and behavior data), is called a grey system. GST has a distinct advantage over the techniques described before, i.e., it can enable the establishment of a

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prediction model using just a small amount of known data. In the context of data starvation, GST is known to be effective and has been widely applied to address real-world problems in the domains of energy management [20], mobile communication [21], instrument measurement [22], stock price analysis [23], and image processing [24], [25]. In addition, Song *et al.* [26] used GST to address the whole-project effort prediction problem in software engineering. They used grey relation analysis (GRA) derived from GST to select more effective feature subsets and similar projects to support a prediction process. Their results showed that GRA can resolve the effort prediction problem with high prediction accuracy. Encouraged by this successful work, we now explore the stage-effort prediction problem using GST in this research.

As we know, there are many project life-cycle models used in the software development domain such as iterative cycles, traditional waterfall models, and so forth. These, in turn, are split into different phases, e.g., feasibility, high-level design, etc. Each of these phases can have complex and, on occasions, ill-defined mappings to physical time units (such as week, month, and quarter). These time units have most generality and are clearly very important in project scheduling. For this reason, in this paper, we focus on software physical time unit, which, for clarity, we refer to as stage-effort prediction. This is in contrast to prediction based on phases, as shown by Mac-Donell and Shepperd [12], that used a seven-phase classification scheme.

We propose a method named GV (GM(1,1) and Verhulst). GV takes full advantage of two grey models of GST—GM(1,1) and Verhulst—to predict future stage effort in light of the law derived from records of prior stage effort, and it can adapt to particular development methodologies automatically by using a novel grey feedback (GFB) mechanism. We validate the method on a large-scale software engineering data repository and obtain very promising results.

The remainder of this paper is organized as follows. In the next section, we discuss related work on, and then present the basic concepts and the prediction principles of the grey models, followed by the introduction of the proposed stage-effort prediction method GV and an example. After that, we describe the datasets and the experimental method we used. The results are then described and discussed, with a concluding discussion presented in the last section.

II. RELATED WORK

Surprisingly, the empirical study of software stage-effort prediction during the development process has received limited attention. The authors are aware of only four studies that have investigated this issue empirically (one being a previous study involving two of the authors [12]). Kulkarni *et al.* [13] employed a form of transformation matrix, referred to as a minimodel, to predict effort for each life-cycle phase. By combining the set of distinct-phase minimodels into a single overall model, it is possible to determine the output measures from the final phase using input measures to the first phase. This method was applied to a military project by Kulkarni *et al.*, and while it appeared to be

potentially effective, prediction accuracy data was not presented. Ohlsson and Wohlin [24] used artifact-based proxies of project scope to augment existing within-project prediction and planning processes. They concluded that the approach has potential in making plans, and deviations from those plans, which are more visible, and that it can increase managers' confidence that predictions are of the right order. MacDonell and Shepperd [12] predicted life-cycle phase effort for 16 software projects using a simple LR method. They used prior-phase effort data to predict the effort needed for subsequent phases in each of the projects. The results showed that the method produced better predictions than those provided by the project managers. A study reported by Abrahamsson *et al.* [27] utilized regression and neural network methods to generate iterative prediction and planning models suitable for projects developed using agile methods. They concluded that the approach improves prediction when compared to whole-project efforts, and that such predictions are stable and convergent, attributes that are essential in terms of effective planning.

A variety of techniques, model inputs, and stage units were used in the studies just described. Compared with these studies, our method—GV—has the following differences or characteristics.

1) GV is capable of predicting project effort for physical time stages. The other studies depend on the use of the life-cycle phase [12], [13], [15] or the iterative development cycle [27].

2) GV uses the inherent trend embedded in the prior stage effort to predict. It does not need the historical data of outputs to build the model. The other studies focused on utilizing the relationships between input and output measures to build the models. MacDonell and Shepperd [12] examined the correlations between phases (e.g., between design and implementation), Ohlsson and Wohlin [15] mapped artifact-based proxies to effort, Abrahamsson *et al.* [27] used LR and neural network methods to express the relationships between inputs and outputs, and Kulkarni *et al.* [13] also constructed a matrix-based model to reflect relationships between inputs and outputs.

3) GV requires project effort itself only as input. MacDonell and Shepperd [12] also used project effort as input. Abrahamsson *et al.* [27] used several estimated predictor variables and the effort of previous iterations as inputs. Ohlsson and Wohlin [15] used their artifact-based proxies as inputs, and Kulkarni *et al.* [13] used object measures (e.g., source lines of code, Ada packages, and data flows).

In short, GV is a novel method to address the problem of stage-effort prediction in software; in principle, however, it would appear to have potential. We now provide an introduction to grey models and to the GV approach.

III. GREY MODEL

GST uses grey models to make predictions. In this section, we will introduce the basis concepts and two grey models used in this paper, respectively.
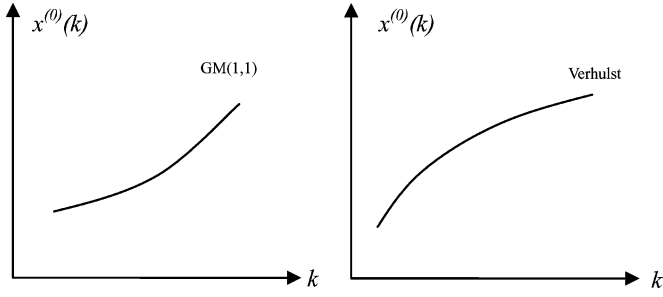
Fig. 1.   Curves of model GM(1,1) and Verhulst.

A. Basis

Generally, the nonnegative raw data sequence has no obvious patterns in uncertain circumstances [19], and it is difficult to find a proper curve to fit it, but after an accumulated generating operation (AGO), the generated sequence will monotonically increase and reflect a strong exponential character. We call this the "accumulated generating grey exponential law." Therefore, we can find an optimum exponential curve to simulate it. The AGO is defined as follows.

Let $X^{(0)}$ be the original nonnegative sequence, the AGO sequence $X^{(1)}$ can be generated as

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \qquad k = 1, 2, \ldots, n. \qquad (1)$$

For example, let $X^{(0)} = (3, 5, 4, 7)$, then $X^{(1)} = (3, 8, 12, 19)$. If the generating exponential character is not obvious, further AGOs may be applied. For returning the data to the original condition, the inverse AGO (IAGO) will be applied. Therefore, AGO and IAGO are a pair of inverse sequence operators. The operation of IAGO is defined as follows:

$$x^{(0)}(1) = x^{(1)}(1)$$
$$x^{(0)}(k+1) = x^{(1)}(k+1) - x^{(1)}(k), \quad k = 1, 2, \ldots, n.$$
$$(2)$$

In GST, the curves used to fit AGO sequences can be represented by differential equations. We refer to these differential equations as grey models. There are many kinds of grey models available according to the different kinds of AGO sequences, such as GM(1,1), Verhulst, DGM, etc. Among them, the GM(1,1) is commonly used to simulate exponential-type sequences; therefore, it is suitable to describe any monotonic increasing procedure. Verhulst is suggested to simulate sequences with saturated trend. The typical curves of these two models [28] are illustrated in Fig. 1, where k denotes the sequence element index and $x^{(0)}(k)$ denotes the sequence element value.

B. GM(1,1) Model

A form of single variable and first-order linear dynamic differential equation

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \qquad (3)$$

where $X^{(1)}$ is the AGO sequence of the original sequence, the parameters a and b are called the development coefficient and grey action quantity, respectively, which can be obtained from the following expression:

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N \qquad (4)$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \cdots & \cdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \qquad (5)$$

$$Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{bmatrix} \qquad (6)$$

and

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}, \qquad k = 2, 3, \ldots, n. \qquad (7)$$

Then, the time response sequence of GM(1,1) can be written as follows:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}, \qquad k = 1, 2, \ldots, n. \qquad (8)$$

where $\hat{x}$ denotes the prediction of x. $\hat{x}^{(0)}(k+1)$ can be obtained by IAGO as

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), \qquad k = 1, 2, \ldots, n. \qquad (9)$$

C. Verhulst Model

The Verhulst model is a form of single variable and second-order differential equation

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b(X^{(1)})^2 \qquad (10)$$

where a and b can be obtained by

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N \qquad (11)$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ \cdots & \cdots \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix} \qquad (12)$$

$$Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \cdots \\ x^{(0)}(n) \end{bmatrix} \qquad (13)$$
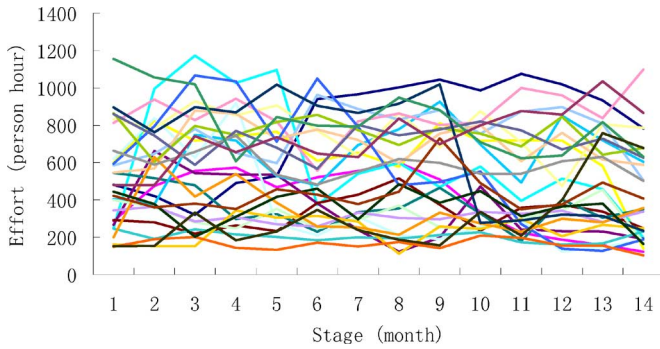
Fig. 2. Curves of some software stage-effort sequences.

Fig. 3. Stage-effort curves of two projects.

and

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}, \qquad k = 2, 3, \dots, n. \quad (14)$$

The time response sequence of Verhulst can be expressed as follows:

$$\hat{x}^{(1)}(k+1) = \frac{ax^{(1)}(0)}{bx^{(1)}(0) + (a - bx^{(1)}(0))e^{ak}},$$
$$k = 1, 2, \dots, n. \quad (15)$$

Note that, in practice, the saturated sequence already has some degree of exponential form, so the original sequence can be regarded as $X^{(1)}$ directly, this means that we need not apply AGO to $X^{(0)}$ to obtain $X^{(1)}$. And $X^{(0)}$ in (13) should be the IAGO sequence of the original sequence.

For the Verhulst model, the prediction values of the original sequence can be obtained from (15).

IV. ANALYSIS OF SOFTWARE STAGE-EFFORT SEQUENCES

The characteristics of software stage-effort sequences[1] are influential in determining the use of appropriate prediction methods. In this section, we introduce some general characteristics of software stage-effort sequences and demonstrate some preliminary analysis of subsequences[2] and sequence sets.[3]

A. Overview

Software stage-effort sequences are records of the software development process. Investigation of stage-effort sequence data used in this study reveals that around 97% of all projects comprise fewer than 24 stages, where a stage represents one month. Therefore, software stage-effort sequences are finite, and generally, quite short in length. In addition, in many cases, software stage-effort sequences do not exhibit a regular shape over the duration of the project, such as the bell-shaped curve or similar, as we might imagine. In fact, they almost have no typical patterns. Fig. 2 provides an illustrative depiction of this scenario, using some of the software stage-effort data analyzed in this study.

[1]Sequence: a series of physical time stage-effort values of a software project.
[2]Subsequence: a subset of a software stage-effort sequence for building prediction model.
[3]Sequence set: a collection of software stage-effort sequences.
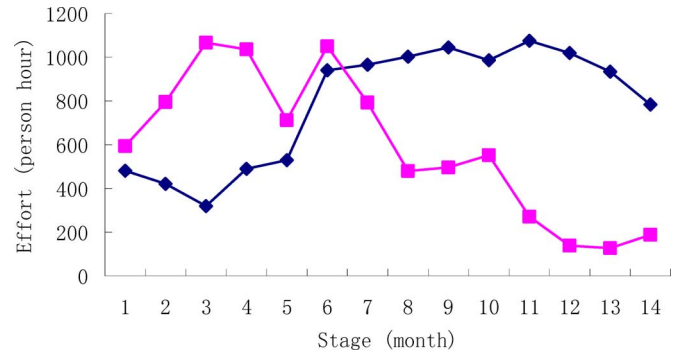
From Fig. 2, we note that at least some of the sequences are very irregular; however, because all of the sequences are plotted on this single graph, it is difficult to identify any consistent patterns within or among them. For ease of observation and comment, we pick out two curves at random from Fig. 2 and show them in Fig. 3. We find that the trend over the whole sequence changes frequently. Within segments of the two sequences, however, the trend demonstrates a degree of stability. In the absence of any consistent whole-sequence patterns, it may be that subsequences can be used to reflect changes in stage effort and can form the basis for the building of accurate effort prediction models.

B. Subsequences

Usually more recent data points are more important in predicting the next data point, so we use recent stage-effort data points to compose each model subsequence. How many stage-effort observations should we use to build a prediction model? In the early stages of the software development process, the number of stage-effort values available is small. A grey model can establish a prediction model with small data samples, but usually more data is helpful. On the other hand, a model that employs many past observations is not always sensitive to data that are liable to frequent and/or significant changes, or can incorporate obsolete values in terms of their relevance to future predictions. Based on these considerations, we use subsequences consisting of the most recent three stage-effort data values.

We find that a subsequence with three elements can be classified into one of the four classes according to its trend and shape. Fig. 4 portrays the four classes of subsequences with characteristics abstracted.

Considering the sequence representations in Fig. 4 in relation to the model types referred to in Fig. 1, we note that curves 1 and 2 are similar to the curves of Verhulst and GM(1,1), respectively. It is also evident that curves 1 and 3, and curves 2 and 4 are symmetrical pairs. If we flip the class 3 or 4 curve in an up/down direction, the curve adopts the form of curve 1 or 2, so they can be dealt with as per curve 1 or 2. Therefore, we can use GM(1,1) or Verhulst to build an appropriate prediction model dynamically according to the types of subsequences identified.
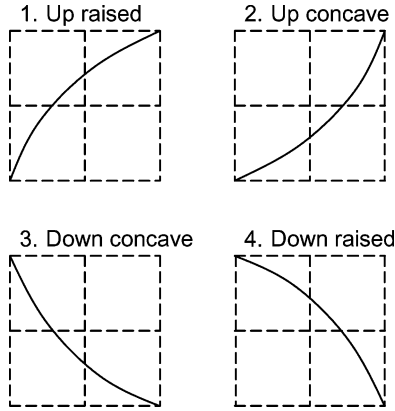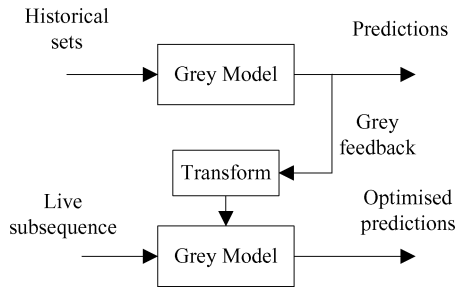
Fig. 4. Four classes of stage-effort subsequences.

Fig. 5. GFB mechanism.

C. Sequence Set

It is more likely that a common law may be found from a sequence set than from a single sequence. Appropriately selected and classified historical projects will be helpful in software effort prediction [4], [10], [26]. Therefore, we group the projects used in this study according to their development methodologies, and hope to find typical patterns in each group. It is reasonable that projects complying with same development process model will have similar stage-effort sequence shape. Unfortunately, however, even within the same group, it may be difficult to find a "standard" shape (as noted in [12]).

Although prior research has failed to find clear laws by observation [29], [30], these works have shown that by using GM(1,1) or Verhulst grey models to make predictions using historical project data grouped by development methodology (or industry, etc.), the mean prediction biases for each group are significantly different. These differences can be regarded as accumulate effects of the particular methodology applying on the predictions. On the other hand, it indicates that the grey models do not differentiate between the methodologies. Adopting a system view, we can regard the stage-effort sequences as inputs, the grey models as a process system, and the biases as outputs. We can then use the mean biases as feedback to adjust the system to obtain more optimal and consistent outputs. The feedback has relations with the development methodologies but its components are not completely clear, so we refer to it as "grey feedback" or GFB. Fig. 5 depicts the mechanism of GFB.

V. PROPOSED SOFTWARE STAGE-EFFORT PREDICTION METHOD GV

A. General Method

In general, GV uses subsequences to build the grey models, and uses sequence sets to obtain GFB. The grey models are then used to make predictions, and GFB is acted on in terms of prediction adjustment, leading to the production of an optimal prediction. Therefore, GV leverages both local (within sequence) and global (sequence set) information to produce predictions. Assume that a live project has a stage-effort sequence, $\{x(1), x(2), \ldots, x(n)\}$, $(n \geq 3)$, GV can use the most recent three stage-effort values $x(n-2)$, $x(n-1)$, and $x(n)$ to predict the effort of stage $n+1$, i.e., $x(n+1)$. The prediction procedure of GV includes the following steps.

Step 1: Construct the model subsequence. Use the most recent three stage-effort data values $x(n-2), x(n-1)$, and $x(n)$ as model subsequence.

Step 2: Smooth the subsequence. Use the modified moving average of order 3 to eliminate unwanted fluctuations of the subsequence (see Section V-B for details).

Step 3: Determine the subsequence type according to the trend (up/down) and shape (raised/concave) (see Section V-C for details).

Step 4: Establish a grey model dynamically according to the subsequence type and produce a preliminary prediction value $\hat{x}(n+1)$ (see Section V-D for details).

In order to facilitate the following description, we represent steps 1–4 as a function

$$\hat{x}(n+1) = \text{Grey\_Predict}(x(n-2), x(n-1), x(n)), \quad n \geq 3. \tag{16}$$

where the input to the function is the subsequence and the output is the prediction value of stage effort for the next stage $n+1$.

Step 5: Adjust the prediction result using the stage-effort adjustment coefficient (SAC) (see Section V-E for details), and then obtain the optimal prediction as

$$\hat{x}_{\text{opt}}(n+1) = \hat{x}(n+1)\text{SAC}. \tag{17}$$

For ease of description, we denote the overall GV prediction procedure (from steps 1–5) by a function

$$\hat{x}_{\text{opt}}(n+1) = GV(x(n-2), x(n-1), x(n)), \qquad n \geq 3. \tag{18}$$

where $\hat{x}_{\text{opt}}(n+1)$ is the final effort prediction value of stage $n+1$. Fig. 6 portrays the details of the general prediction procedure of GV.

B. Modified Moving Average of Order 3

Smooth transformation can reduce noise-based fluctuations in the sequences. A common transformation method is the weighted moving average of order k [31]

$$\frac{\omega_1 y_1 + \omega_2 y_2 + \cdots + \omega_k y_k}{\omega_1 + \omega_2 + \cdots + \omega_k}, \frac{\omega_2 y_2 + \omega_3 y_3 + \cdots + \omega_{k+1} y_{k+1}}{\omega_2 + \omega_3 + \cdots + \omega_{k+1}},$$

$$\frac{\omega_3 y_3 + \omega_4 y_4 + \cdots + \omega_{k+2} y_{k+2}}{\omega_3 + \omega_4 + \cdots + \omega_{k+2}}, \cdots \tag{19}$$
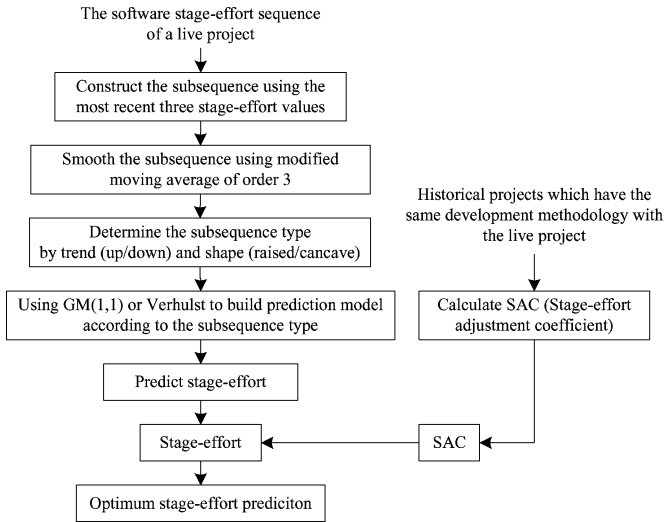
Fig. 6. Prediction procedure of GV.

Fig. 7. Modified moving average of order 3.

Fig. 8. Determination of subsequence type procedure.

where $Y = (y_1, y_2, y_3, \ldots)$ is a given sequence and ω_i is the weight. After transformation, $k-2$ head and tail data are discarded. This would likely raise problems in software stage-effort prediction as, in general, the stage-effort data of a software project are limited, particularly, in the early stages of development. The aforementioned transformation makes small data samples even smaller, perhaps of insufficient size to construct a prediction model. Therefore, we modify (19) by reserving the head and tail data. For example, the weighted moving average of order 3 used in this paper can be treated as

$$\frac{3y_1 + y_2}{4}, \ldots, \frac{y_{k-1} + 2y_k + y_{k+1}}{4}, \ldots, \frac{y_{n-1} + 3y_n}{4}. \quad (20)$$

Specifically, for a sequence of three elements (y_1, y_2, y_3), the transformed sequence is

$$\frac{3y_1 + y_2}{4} \qquad \frac{y_1 + 2y_2 + y_3}{4} \qquad \frac{y_2 + 3y_3}{4}. \quad (21)$$

The three elements can form a folding line. The basic shape of the folding line will not change after the transformation, and a straight line will still be a straight line after the transformation. Fig. 7 illustrates the original and transformed sequences where the x-axis denotes the indexes of the elements and the y-axis denotes the values. The circles represent the original data and the squares represent the transformed values.

From Fig. 7, we can see that the transformed sequence is more regular than the original but that the original trend is preserved.

C. Determination of Subsequence Type

We assume that the subsequence is composed of (y_1, y_2, y_3). The type determination procedure includes two stages. Fig. 8 shows in pseudocode the detailed determination procedure of subsequence type.

First, classify the subsequence into up or down classes by comparing y_1 with y_3. If $y_3 > y_1$, we say that the subsequence has a total up trend, if $y_3 < y_1$, we say it has a total down trend, if $y_3 = y_1$ (horizontal), we regard it as a down trend (steps 1 and 8, respectively, in Fig. 8).
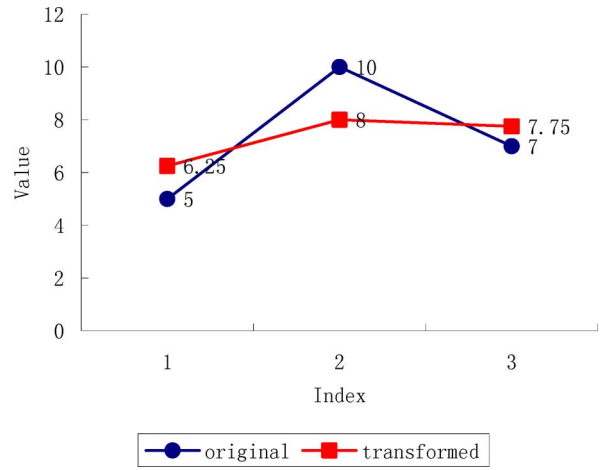
We then classify the subsequence into raised or concave classes by comparing y_2 with the median of y_1 and y_3, i.e., $(y_1 + y_3)/2$: 1) for an up class subsequence, if $y_2 > (y_1 + y_3)/2$, we say the subsequence is raised, type is 1 (steps 3 and 4), if $y_2 < (y_1 + y_3)/2$, we say it is concave, type is 2. When $y_2 = (y_1 + y_3)/2$, it is a straight line, we include it in the concave class (steps 5 and 6) and 2) for a down class subsequence (including $y_3 = y_1$), if $y_2 < (y_1 + y_3)/2$, we say the subsequence is concave, type is 3 (steps 10 and 11), if $y_2 > (y_1 + y_3)/2$, we say it is raised, type is 4. When $y_2 = (y_1 + y_3)/2$, it is a straight line, we include it in the raised class (steps 12 and 13).

D. Prediction Model

GV establishes models dynamically according to the types of stage-effort subsequence. Considering the subsequence representations in Fig. 4 in relation to the model types referred to in Fig. 1, we note that curve 1 is up-raised, i.e., a saturated sequence, so a Verhulst model can be used to represent it. Curve 2 is up-concave, i.e., an exponential sequence, and thus, is theoretically suitable to be predicted by GM(1,1). Curves 3 and 4 are down sequences that cannot use GM(1,1) or Verhulst directly,
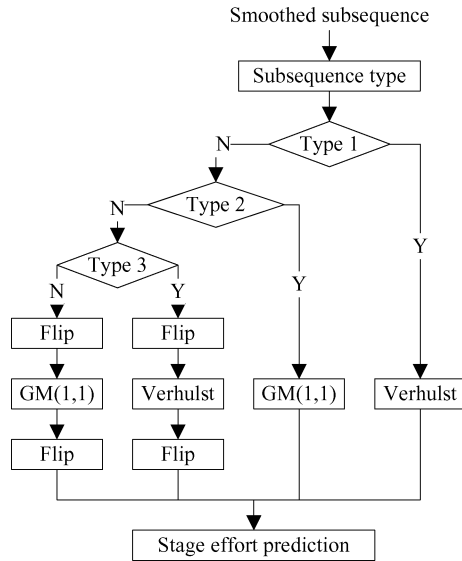
Fig. 9. Model and prediction procedure.

TABLE I
TYPES AND TREATMENTS OF STAGE-EFFORT SUBSEQUENCES

Type	Class of original subsequence	Suitable model
1	Saturated	Verhulst
2	Exponential	GM(1,1)
3	Saturated after up flipping	Verhulst
4	Exponential after up flipping	GM(1,1)

because GM(1,1) and Verhulst require the sequences to have a total upward trend for accurate predictions. Therefore, we must first convert these curves to up-style sequences. In fact, as noted, it is clear that curves 1 and 3, and curves 2 and 4 are symmetrical pairs. If we flip the class 3 or 4 curve in an up/down direction, it adopts the form of curve 1 or 2. We can then use GM(1,1) or Verhulst to model and predict the curves. After predicting, we need to flip again to ensure that the models match the original sequences. The detailed procedure is shown in Fig. 9. In summary, the classes and suitable models for various types of original data subsequences are listed in Table I.

E. Obtaining SAC and GFB

SAC forms a bridge between the grey models and particular development methodologies. Using it, GV can fit the particular stage-effort sequences better and obtain more accurate results. In prior research [29], [30], a bias correction method has been used, but it tends to overfit. In this study, we make improvements based on the prior work, and propose a novel adjustment coefficient SAC as follows:

$$\text{SAC} = \frac{e^{\text{GFB}/N}}{1 - \text{GFB}} \qquad (22)$$

where N is the length of a GV model subsequence; in this study, it is equal to three. GFB is intended here as a form of GFB pertaining to particular development methodologies (see Section IV-C for details). It can be estimated using the mean biases of predictions of historical stage-effort sequences.

Function: GetGFB, obtain the *GFB* (grey feedback) from a set of software
           stage-effort sequences.
Input: a set of stage-effort sequences.
Output: *GFB*
1) $j = 1$;
2) **for** each stage-effort sequence $\{x(1), x(2),..., x(n)\}$, $(n \geq 3)$
3)   **for** each i from 3 to n-1
4) $\hat{x}(i+1) = \text{Grey\_Predict}(\ x(i-2),\ x(i-1),\ x(i)\ )$;
5) $Bias(j) = \dfrac{x(i+1) - \hat{x}(i+1)}{x(i+1)}$;
6) $j = j + 1$;
7)   **end for**
8) **end for**
9) $GFB = \dfrac{1}{j-1}\sum\limits_{k=1}^{j-1} Bias(k)$;
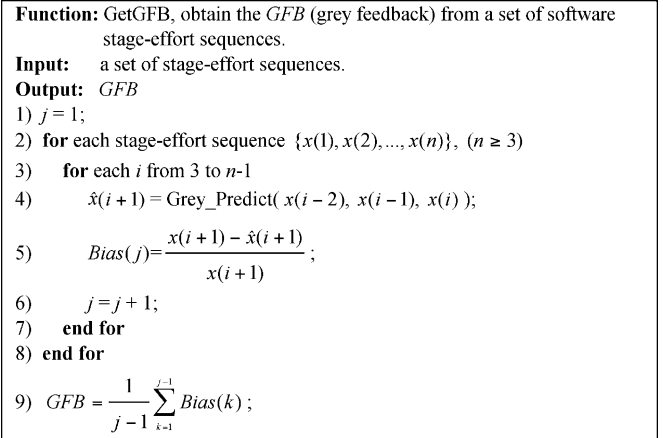
Fig. 10. GFB obtaining function.

To obtain GFB, we need a set of historical projects that comply with certain development methodologies. Each project has a stage-effort sequence and is regarded as a live project. First, we use (16) to make predictions stage by stage for each project, then compute the bias of each prediction. This process iterates until all projects are processed. Finally, we average all the biases and assign the result to GFB. The procedure for obtaining GFB is summarized in Fig. 10.

F. Example

We now demonstrate the complete prediction procedure for a "live" project using the GV method. The initial condition is: a live project that has been in progress for three stages (three months), stage-effort values (in person-hours) of 260, 101, and 450, and SAC = 0.76. The prediction procedure of the effort needed to complete the fourth stage (the fourth month) is as follows.

Step 1: Construct the model subsequence using the three most recent stage-effort values. The subsequence is $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3)) = (260, 101, 450)$.

Step 2: Smooth the subsequence. Use (21) to transform the subsequence into: $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3)) = (220, 228, 363)$.

Step 3: Determine the subsequence type. According to Section V-C, because $x^{(0)}(1) < x^{(0)}(3)$ and $x^{(0)}(2) < (x^{(0)}(1) + x^{(0)}(3))/2$, so $X^{(0)}$ is an up-concave sequence, belongs to type 2, and according to Table I, the suitable prediction model is GM(1,1).

Step 4: Build the prediction model. According to the building procedure of GM(1,1) (see Section III-B for details), first apply AGO on $X^{(0)}$ and obtain the AGO sequence: $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), x^{(1)}(3)) = (220, 448, 811)$. Then, we can get $Z^{(1)}$, B, and Y_N as

$$Z^{(1)} = \begin{bmatrix} z^{(1)}(2) \\ z^{(1)}(3) \end{bmatrix} = \begin{bmatrix} \dfrac{x^{(1)}(2) + x^{(1)}(1)}{2} \\ \dfrac{x^{(1)}(3) + x^{(1)}(2)}{2} \end{bmatrix} = \begin{bmatrix} 334 \\ 629.5 \end{bmatrix}$$

$$Y_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \end{bmatrix} = \begin{bmatrix} 228 \\ 363 \end{bmatrix}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \end{bmatrix} = \begin{bmatrix} -334 & 1 \\ -629.5 & 1 \end{bmatrix}$$

the arguments a and b can be obtained by (4)

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_N = \begin{bmatrix} -0.4569 \\ 75.4112 \end{bmatrix}.$$

Therefore, we can get the prediction model of $X^{(1)}$ by (8)

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}$$

$$= 385.0497 e^{0.4569k} - 165.0497.$$

Let $k = 1, 2, 3$, we can get $\hat{X}^{(1)} = (\hat{x}^{(1)}(2), \hat{x}^{(1)}(3), \hat{x}^{(1)}(4)) = (443, 795, 1351)$. Note that $\hat{x}^{(1)}(4)$ corresponds to stage 4 (the stage for which we are predicting).

Step 5: Make the preliminary prediction. According to (9), we can get the preliminary prediction of stage 4: $\hat{x}^{(0)}(4) = \hat{x}^{(1)}(4) - \hat{x}^{(1)}(3)) = 1351 - 795 = 556$.

Step 6: Apply SAC to the preliminary prediction to obtain the optimum prediction. The final prediction value is: $556 \times \text{SAC} = 556 \times 0.76 = 422$.

Therefore, the prediction effort for the fourth stage of this project is 422 person-hours. After one further month, we obtain the real-effort value of the fourth month, and then, the fifth stage-effort value can be predicted by the known effort of stages 2, 3, and 4. The procedure is executed like this until the project completes overall. Note that SAC is obtained from historical projects, but if no historical projects are available, then let SAC = 1.

VI. EXPERIMENTS AND ANALYSIS

A. Data Sources

The data used in this research are drawn from a large-scale software engineering data repository.[4] The data contain information relating to around 1900 projects undertaken between 2000 and 2004. The projects come from around 30 countries or areas. The main contributing countries are: the United States, Australia, Sweden, Canada, New Zealand, and the United Kingdom. The application domains range from aerospace, financial, manufacturing, medicine, traffic, and business. The project data in the repository are composed of project properties and effort data. The static properties, such as development methodology, industry, development team, project type, experience levels, etc., are collected once at the beginning of each project. The databases, platforms, computer-aided software engineering (CASE) tools, etc., are recorded as changes in each occur. Each project is described in terms of a set of concurrent activities that comply with a certain methodology. The effort for

[4]We regret that the data set is subject to a non-disclosure agreement due to its commercially sensitive nature.

TABLE II
DISTRIBUTION OF PROJECTS

Dataset	Description	Count
1	Combined Dev+ Life Cycle Modules	864
2	App Development Overlapping Waterfall	567
3	SSP−Software Engineering and Release	390
4	Application Implementation	45

TABLE III
DESCRIPTIVE STATISTICS OF PROJECT STAGE EFFORT (IN PERSON HOURS PER MONTH)

Dataset	Mean	Std. Dev	Min	Max	Median
1	729.17	1701.96	0	37765.50	331.25
2	941.03	3604.13	0	69233.00	312.50
3	437.74	737.15	0	7892.50	194.00
4	649.38	1378.09	0	19845.00	260.00

each live development activity is recorded monthly. To obtain a general view, we employ the monthly effort of a project (instead of its composite low-level activities) as the figure of interest in each stage in the experiments of this study. The former is the sum of the latter for each month.

As stated previously, GV can adjust itself to suit particular development methodologies, so we organized the projects into four datasets drawn from the underlying dataset according to development methodology. Tables II and III summarize the real raw data stored in the repository and show that the number of projects per methodology ranges from 45 to 864, the monthly effort ranges from 0 to 69 233 person hours. Therefore, the datasets are large (by software engineering standards) and methodologically diverse. It has long been acknowledged that the absence of systematic historical data is a significant obstacle in the software effort prediction domain [17], [32], with experiments and evaluations having to rely on small data samples [6]. The datasets used in this study enable us to evaluate and calibrate the proposed method with data from large numbers and various kinds of projects, and should, therefore, lead to more robust results and reliable conclusions.

B. Experimental Methods

1) General Method: The purpose of the experiments is to evaluate the prediction performance of GV. First, we preprocess the four datasets (see Section VI-B2 for details), then for each of the four datasets, we systematically extract five pairs of the training and test datasets, and obtain a total of 20-pair training–test datasets. We obtain GFB and SAC on each training set, and evaluate GV on the test set (see subsection Section VI-B3 for details). Finally, we use a LR method and the Kalman filter (KF) method (see Section VI-B4 for details) as benchmarks to compare against the performance of GV.

2) Data Preprocessing: Inspection of the datasets reveals that there is a lot of noisy and inconsistent data. We preprocess the data as follows.

Delete the extreme[5] cost data values from the stage-effort sequences. These values are too small or too big when compared with the other effort values. The stages with such extreme values are abnormal stages and could not be predicted using a generally useful model. For instance, the minimum value of stage effort is zero, and this is considered an extreme value (see Table III).

3) Validation Method: Cross-validation is a method for estimating generalization error based on "resampling" [33]. We use a five-fold cross-validation strategy as the validation approach. In five-fold cross-validation, the dataset $\mathscr{D}$ is randomly partitioned into five mutually exclusive subsets, $\mathscr{D}_1, \mathscr{D}_2, \ldots, \mathscr{D}_5$, each of approximately equal size. The inducer is trained and tested five times. Each time $t \in \{1, 2, \ldots, 5\}$, the subset $\mathscr{D}_t$ is reserved as the test set, and the remaining subsets $\mathscr{D} \ominus \mathscr{D}_t$[6] are used as training set.

We obtain SAC from the training set (see Section V-E for details) and evaluate GV on the test set. When evaluating, we regard the projects in the test set as live projects, i.e., for each project with stage-effort sequence $\{x(1), x(2), \ldots, x(n)\}$, $(n \geq 3)$, we use $\{x(k-2), x(k-1), x(k)\}$, where $3 \leq k \leq n-1$, as the input subsequence, and use (18) to obtain the prediction value $\hat{x}_{\text{opt}}(n+1)$. For a project with n stages, we can make $n-3$ predictions. For each prediction, we compare the prediction value with the actual value to obtain evaluation results. In this study, we use the bias, mean magnitude of relative error (MMRE), and median MRE (MdMRE) as evaluation measures.

The Bias establishes whether models are biased and tend to over or under prediction. The Bias is defined as follows:

$$\text{Bias}_i = \frac{\varepsilon_i - \hat{\varepsilon}_i}{\varepsilon_i} \tag{23}$$

where $\hat{\varepsilon}$ is the prediction value of actual effort ε.

The magnitude of relative error (MRE) is another common criterion for evaluating software effort prediction methods. For a prediction i, the corresponding MRE_i is defined as follows:

$$\text{MRE}_i = |\text{Bias}_i|. \tag{24}$$

By averaging MRE_i over multiple predictions n, MMRE is obtained as

$$\text{MMRE} = \frac{1}{n} \sum_{i=1}^{n} \text{MRE}_i. \tag{25}$$

MMRE is the most frequently used criterion for evaluating software effort prediction methods. However, it is known to be sensitive to individual predictions with excessively large MREs. We, therefore, also use MdMRE for the n predictions, which is less sensitive to extreme values as another measure. For both MMRE and MdMRE, a higher value means lower prediction accuracy.

4) Benchmark Methods: MacDonell and Shepperd employed a simple LR method in [12], and because there is no other work that can be compared, we use LR as a benchmark.

[5]If a value is very large or small compared with its adjacent values in the sequence, then we call it an extreme.

[6]The notation $\mathscr{D} \ominus \mathscr{D}_t$ means set $\mathscr{D}$ minus set $\mathscr{D}_t$.

TABLE IV
MMRE OF GV, LR, AND KF, AND GV'S IMPROVEMENT UPON KF AND LR
WITH DIFFERENT DATASETS

Dataset	MMRE (%)			GV's MMRE improvement (%)	
	GV	KF	LR	upon KF	upon LR
1	62.60	80.35	111.29	28.35	77.78
2	61.44	84.98	131.18	38.31	113.51
3	61.52	95.69	134.64	55.54	118.86
4	54.48	77.95	81.63	43.08	49.83

TABLE V
MdMRE OF GV, LR, AND KF, AND GV'S IMPROVEMENT UPON KF AND LR
WITH DIFFERENT DATASETS

Dataset	MdMRE (%)			GV's MdMRE improvement (%)	
	GV	KF	LR	upon KF	upon LR
1	46.96	59.02	73.87	25.68	57.30
2	44.17	53.89	74.59	22.01	68.87
3	49.81	67.59	85.43	35.70	71.51
4	42.04	57.01	64.31	35.61	52.97

However, as LR often suffers from a leverage effect of outliers [17], we complement it with a further approach. The KF method [34] is a well-known series prediction method that can deal with noise and outliers robustly [35], so we also use the KF as another benchmark method and determine its arguments from the sequences in the training set (the training and test sets KF used are same as those used with GV, see Section VI-B3 for details). For fairness, we use the same datasets and subsequences for the three approaches.

As many statistical techniques that deal with the prediction of time series data require large samples (i.e., long sequences), so they are not suitable to be benchmarks here. For example, effective fitting of Box–Jenkins models (often called autoregressive integrated moving average (ARIMA) models) [36] typically requires at least 50 observations [37]. This is too many for most software project stage-effort sequences and cannot be satisfied here.

C. Experimental Results

We conduct the experiments using GV (the GV method), KF (the KF method), and LR (the simple LR method) on the four datasets. Tables IV–VI report the accuracy of the respective methods in terms of MMRE, MdMRE, Bias, and the improvements of GV upon KF and LR.

From Tables IV–VI, we observe that the MMREs, MdMREs, and Biases of GV are superior to those of KF and LR for all four datasets. Further, GV's prediction accuracies across all datasets exhibit smaller differences. This indicates that GV has better consistency and stability on different development methodologies. The small bias values also imply that GV can cope with the differences in development methodologies to obtain optimal results. We also observe that compared with KF and LR, GV's MMREs are lower by at least 28% and 50%, respectively,

TABLE VI
BIAS OF GV, LR, AND KF, AND GV'S IMPROVEMENT UPON KF AND LR WITH DIFFERENT DATASETS

Dataset	Bias (%)			GV's Bias improvement (%)	
	GV	KF	LR	upon KF	upon LR
1	-0.76	-25.73	-29.04	3285.53	3721.05
2	0.18	-35.83	-34.49	19805.56	19061.11
3	3.22	-40.42	-50.31	1155.28	1462.42
4	5.85	-19.73	-20.98	237.26	258.63

TABLE VII
PERCENTILES OF ABSOLUTE RESIDUALS OF PREDICTION ACCURACY FOR GV, KF, AND LR WITH ALL DATASETS

dataset	Method	Percentiles						
		5	10	25	50	75	90	95
1	GV	9.4	20.7	57.7	159.1	363.8	764.8	1160.2
	KF	13.0	24.5	64.7	168.7	409.5	858.4	1302.9
	LR	12.8	27.9	75.4	182.4	404.1	804.2	1182.3
2	GV	8.2	18.6	52.7	153.2	375.6	788.8	1462.0
	KF	12.0	24.4	69.4	178.9	413.3	877.8	1465.8
	LR	12.8	28.2	74.0	186.9	417.4	840.3	1480.4
3	GV	4.2	10.4	34.6	94.4	228.7	519.3	801.0
	KF	6.7	17.9	45.6	121.7	286.2	607.5	934.3
	LR	7.1	16.3	47.8	128.6	292.8	584.0	901.9
4	GV	5.3	11.0	34.7	100.4	270.0	540.6	688.0
	KF	8.5	14.3	37.4	104.8	315.6	588.0	861.6
	LR	5.6	12.5	40.5	106.5	265.6	599.9	914.0

TABLE VIII
ONE-TAILED WILCOXON MATCHED-PAIRS SIGNED RANKS TEST OF MMREs OF GV VERSUS KF AND GV VERSUS LR ON ALL DATASETS

Dataset	p-value	
	GV vs KF	GV vs LR
1	.000	.000
2	.000	.000
3	.000	.000
4	.000	.000

From Table VIII, we observe that all p-values are less than 0.001. This means that the MMREs of GV are significantly lower (and, therefore, better) than those of KF and LR.

To summarize, in this study, GV outperforms KF and LR on all four datasets and demonstrates considerable potential. The reasons lie in the following.

1) The AGO procedures of grey models make the original stage-effort sequences more regular, and thus, easier to fit.
2) The subsequence type recognition procedures makes GV flexible, which means GV can capture the various changing trends and further take full advantage of two grey models.
3) The capability of using GFB from historical projects means GV can utilize global domain knowledge to pilot local predictions along correct directions, avoiding reliance on just small local datasets and maximizing the capability of the method.

VII. CONCLUSION

Dynamic software project stage-effort prediction facilitates the evaluation of potential effort problems, and could provide early warning information, thus ensuring that projects are completed within (possibly adjusted) schedules and budgets. In this paper, we have proposed a novel approach of using grey models of grey system theory to address the software stage-effort prediction problem during the development process. The proposed method can predict the future stage effort using the effort of three most recent continuous stages and can suit particular development methodologies by using a novel GFB mechanism.

Our experiments have been conducted on a large-scale software engineering data repository split into four datasets based on development methodology. Because there is no other work that can be compared, we employed the KF method and the LR method as benchmarks. The results show that the proposed GV method outperforms KF and LR in terms of MMRE, MdMRE, and Bias for all datasets used.

Finally, we pose the question of generalization. The datasets are drawn from a repository of medium to large projects from an international software house. They cover a range of application areas such as commerce, government information systems, defense, and retail. Of course, different organizations may run projects and collect data in different ways. Therefore, it would be interesting to see replication of this study in different environments. Nevertheless, this is an encouraging result and shows that the method has considerable potential.

MdMREs are lower by at least 22% and 53%, respectively, and GV shows very good performance with respect to Bias, with values that are lower by at least 237% and 259% than KF and LR, respectively.

In statistics, percentiles are used to describe characteristics of distributions. Here, we used percentiles to explore the distribution of the absolute residuals of prediction accuracy for the three methods with four datasets. Table VII contains the results. From it, we can find that GV has less values than KF and LR for the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles with all four datasets except for the 75th percentile of dataset 4, LR has a less value. This reveals that GV outperformed both KF and LR.

Although we have made some observations based on the data presented in the tables before, to rigorously compare the differences between prediction accuracy between the three methods, we need to perform statistical significance testing. The MMREs of the three methods do not follow a normal distribution, so we use one-tailed Wilcoxon matched-pairs signed ranks tests to examine if there exist significant improvements on the MMREs of GV over the other two methods. Table VIII gives the testing results.

REFERENCES

[1] M. Jørgensen and M. Shepperd, "A systematic review of software development cost estimation studies," *IEEE Trans. Softw. Eng.*, vol. 33, no. 1, pp. 33–53, Jan. 2007.

[2] M. Shepperd, "Software project economics: A roadmap," in *Proc. Future Softw. Eng. (FOSE 2007)*, Minneapolis, MN, 2009, pp. 304–315.

[3] B. W. Boehm, *Software Engineering Economics*. Englewood Cliffs, NJ: Prentice-Hall, 1981.

[4] M. Shepperd and C. Schofield, "Estimating software project effort using analogies," *IEEE Trans. Softw. Eng.*, vol. 23, no. 11, pp. 736–743, Nov. 1997.

[5] K. Moløkken and M. Jørgensen, "A review of surveys on software effort estimation," in *Proc. 2003 Int. Symp. Empirical Softw. Eng. (ISESE 2003)*, Rome, Italy, pp. 223–230.

[6] L. C. Briand, T. Langley, and I. Wieczorek, "A replicated assessment and comparison of common software cost modeling techniques," in *Proc. 22th Int. Conf. Softw. Eng. (ICSE 2000)*, Limerick, Ireland, pp. 377–386.

[7] L. C. Briand, K. E. Emam, D. Surmann, I. Wieczorek, and K. D. Maxwell, "An assessment and comparison of common software cost estimation modeling techniques," in *Proc. 21st Int. Conf. Softw. Eng. (ICSE 1999)*, Los Angeles, CA, pp. 313–323.

[8] M. Jørgensen, "How much does a vacation cost? Or what is a software cost estimate?" *ACM SIGSOFT Softw. Eng. Notes*, vol. 28, no. 6, p. 5, Nov. 2003.

[9] A. Idri, T. M. Khoshgoftaar, and A. Abran, "Can neural networks be easily interpreted in software cost estimation?," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE 2002)*, Hawaii, USA, pp. 1162–1167.

[10] B. W. Boehm, B. K. Clark, E. Horowitz, A. W. Brown, D. J. Reifer, S. Chulani, R. Madachy, and B. Steece, *Software Cost Estimation with Cocomo II*. New York: Prentice-Hall, 2000.

[11] G. R. Finnie and G. E. Wittig, "AI tools for software development effort estimation," in *Proc. Int. Conf. Softw. Eng.: Edu. Practice (SE:EP 1996)*, Dunedin, New Zealand, pp. 346–353.

[12] S. MacDonell and M. Shepperd, "Using prior-phase effort records for re-estimation during software projects," in *Proc. 9th IEEE Int. Softw. Metrics Symp. (METRICS 2003)*, Sydney, Australia, pp. 73–86.

[13] A. Kulkarni, J. B. Greenspan, D. A. Kriegman, J. J. Logan, and T. D. Roth, "A generic technique for developing a software sizing and effort estimation model," in *Proc. 12th Int. Comput. Softw. Appl. Conf. (COMPSAC 1988)*, Chicago, USA, pp. 151–161.

[14] N. E. Fenton and S. L. Pfleeger, *Software Metrics, A Rigorous and Practical Approach*, 2nd ed. Boston, MA: PWS-Kent, 1997.

[15] M. C. Ohlsson and C. Wohlin, "An empirical study of effort estimation during project execution," in *Proc. 6th Int. IEEE Softw. Metrics Symp. (METRICS 1999)*, Boca Raton, USA, pp. 91–98.

[16] A. Rainer and M. Shepperd, "Re-planning for a successful project schedule," in *Proc. 6th IEEE Int. Softw. Metrics Symp. (METRICS 1999)*, Boca Raton, USA, pp. 72–81.

[17] G. Liebchen, B. Twala, M. Shepperd, M. Cartwright, and M. Stephens, "Filtering, robust filtering, polishing: Techniques for addressing quality in software data," in *Proc. 1st Int. Symp. Empirical Softw. Eng. Meas. (ESEM 2007)*, Madrid, Spain, pp. 99–106.

[18] M. Jørgensen and S. Grimstad, "Avoiding irrelevant and misleading information when estimating development effort," *IEEE Softw.*, vol. 25, no. 3, pp. 78–83, May/Jun. 2008.

[19] J. Deng, "Control problems of grey systems," *Syst. Control Lett.*, vol. 1, no. 5, pp. 288–294, 1982.

[20] T. H. M. El-Fouly, E. F. El-Saadany, and M. M. A. Salama, "Grey predictor for wind energy conversion systems output power prediction," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1450–1452, Aug. 2006.

[21] S. L. Su, Y. C. Su, and J. F. Huang, "Grey-based power control for DS-CDMA cellular mobile systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 6, pp. 2081–2088, Nov. 2000.

[22] K. Lin and B. Liu, "A gray system modeling approach to the prediction of calibration intervals," *IEEE Trans. Instrum. Meas.*, vol. 54, no. 1, pp. 297–304, Feb. 2005.

[23] Y. F. Wang, "On-demand forecasting of stock prices using a real-time predictor," *IEEE Trans. Knowl. Data Eng.*, vol. 15, no. 4, pp. 1033–1037, Jul./Aug. 2003.

[24] R. C. Luo, T. M. Chen, and K. L. Su, "Target tracking using a hierarchical grey-fuzzy motion decision-making method," *IEEE Trans. Syst., Man, Cybern. A: Syst., Humans*, vol. 31, no. 3, pp. 179–186, May 2001.

[25] C. Jau-Ling and C. Pei-Yin, "An efficient gray search algorithm for the estimation of motion vectors," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 31, no. 2, pp. 242–248, May 2001.

[26] Q. Song, M. Shepperd, and C. Mair, "Using grey relational analysis to predict software effort with small data sets," in *Proc. 11th IEEE Int. Softw. Metrics Symp. (METRICS 2005)*, Rome, Italy, p. 35.

[27] P. Abrahamsson, R. Moser, W. Pedrycz, A. Sillitti, and G. Succi, "Effort prediction in iterative software development processes—Incremental versus global prediction models," in *Proc. 1st Int. Symp. Empirical Softw. Eng. Meas. (ESEM 2007)*, Madrid, Spain, pp. 344–353.

[28] S. Liu, Y. Dang, and Z. Fang, *The Grey System Theory and Applications (in Chinese)*. Beijing, China: Science Press, 2004.

[29] Y. Wang, Q. Song, and J. Shen, "Grey learning based software stage-effort estimation," in *Proc. 6th Int. Conf. Mach. Learning Cybern. (ICMLC 2007)*, Hong Kong, China, pp. 1470–1475.

[30] Y. Wang, Q. Song, and J. Shen, "Grey prediction based software stage-effort estimation," *Wuhan Univ. J. Nat. Sci.*, vol. 12, no. 5, pp. 927–931, 2007.

[31] J. W. Han and M. Kamber, *Data Mining: Concepts and Techniques*. San Fransisco, CA: Morgan Kaufmann, 2001.

[32] M. Shepperd and M. Cartwright, "Predicting with sparse data," *IEEE Trans. Softw. Eng.*, vol. 27, no. 11, pp. 987–998, Nov. 2001.

[33] J. Shao and D. Tu, *The Jackknife and Bootstrap*. New York: Springer-Verlag, 1995.

[34] R. E. Kalman, "A new approach to linear filtering and prediction problems," *Trans. ASME J. Basic Eng.*, vol. 82, no. 1, pp. 35–45, 1960.

[35] H. Weiming, T. Tieniu, W. Liang, and S. Maybank, "A survey on visual surveillance of object motion and behaviors," *IEEE Trans. Syst., Man, Cybern. C, Appl. Rev.*, vol. 34, no. 3, pp. 334–352, Aug. 2004.

[36] P. Brockwell and R. Davis, *Introduction to Time Series and Forecasting*, 2nd ed. New York: Springer-Verlag, 2002.

[37] C. Chatfield, *The Analysis of Time Series*, 5th ed. New York: Chapman & Hall, 1996.

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